Solutions

6.2: Cardinality

Definition 1. If A is a finite set, then its cardinality is the number of elements in A. We denote the cardinality by $n(A)$ or $|A|$.

Example 1.

(a) Let $S = \{a, b, c\}$. Then $|S| = 3$.

(b) Let A be the set of outcomes when rolling one die. Then $n(A) = 6$.

Cardinality of a Union

Question 1. How can we calculate $n(A \cup B)$ if we know $n(A)$ and $n(B)$? Give an initial guess and then try the next example.

An initial guess may be something like $n(A \cup B) = n(A) + n(B)$.

However, this is incorrect. After completing example 2, you should see that the correct formula is

Example 2. Let $A = \{a, b, c\}$ and $B = \{b, c, d\}$. Find $n(A)$, $n(B)$ and $n(A \cup B)$. Was your guess correct? If not, what needs to be changed?

$$\boxed{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$$

$n(A) = 3$, $n(B) = 3$, $A \cup B = \{a, b, c, d\}$

So that $n(A \cup B) = 4$. Also $n(A \cap B) = 2$ so the boxed formula is correct.

Cardinality of a Complement

Question 2. Let S be a finite universal set containing A. Give an initial guess for $n(A')$.

$n(A') = n(S) - n(A)$.

Example 3. Let S be the possible outcomes of rolling a single die. Let A be the subset of S which contains all rolls strictly greater than 2. Find $n(A)$, $n(S)$ and $n(A')$. Was your guess correct?

$A = \{3, 4, 5, 6\}$ so $n(A) = 4$. $S = \{1, 2, 3, 4, 5, 6\}$ so $n(S) = 6$.

$A' = \{1, 2\}$ so $n(A') = 2 = 6 - 4$ as desired.